Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 27, 2003*
Accepted: *February 6, 2003*

Perturbative and non-perturbative aspects of pure $\mathcal{N} = 1$ super Yang-Mills theory from wrapped branes

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Abstract: The Maldacena-Nuñez solution is generalized to include a number of integration constants, one of which controls the resolution of the singularity of the wrapped D5-brane background. Some features of the dual pure $\mathcal{N} = 1$ super Yang-Mills (SYM) theory are calculated, amongst which the gluino condensate, the beta function of the gauge coupling and a brane probe potential, which is related to the Veneziano-Yankielowicz effective potential. Each integration constant has a precise meaning in the dual SYM theory, e.g., the amount of non-perturbative SYM physics captured by the gravity configuration is described by the singularity resolution parameter.

Keywords: Nonperturbative Effects, D-branes, Brane Dynamics in Gauge Theories.

JHEP02(2003)013

Contents

1. Introduction and summary

The possibility of studying super Yang-Mills (SYM) theories using their gravity duals has been a surprising manifestation of 't Hooft's old idea that gauge theories have a string theoretical microscopic origin [1]. After the success of the AdS/CFT correspondence [2, 3, 4] (see also the recent lecture notes [5, 6]) in describing the large-N limit of (super) conformal SYM theories (e.g., $\mathcal{N} = 4$ SYM theory) by asymptotically AdS super gravity (SUGRA) backgrounds, a systematic formulation of a more general "gauge/gravity" duality describing also non-conformal SYM theories (or gauge theories with fewer supersymmetries) is still an outstanding problem. A huge amount of work has recently been devoted to the study of specific cases of this duality, and much relevant information of the SYM theories has been extracted from their SUGRA duals. One way to reduce supersymmetry is to consider SUGRA backgrounds generated by branes wrapping supersymmetric cycles [7, 8]. Thus, the identification of such a SUGRA dual of pure $\mathcal{N} = 1$ SYM theory by Maldacena and Nuñez (MN) [9] was a major achievement and has spurred a lot of activity.[1] For a list of references to other known cases of the gauge/gravity correspondence, we refer the reader to [11].

[1] Another dual of pure $\mathcal{N} = 1$ SYM theory is the Klebanov-Strassler solution [10] describing fractional 3-branes at the apex of a deformed conifold.

JHEP02(2003)013

The MN solution was originally found as a BPS magnetic monopole by Chamseddine and Volkov [12, 13], and since MN's work it has been the subject of a number of articles. A qualitative analysis of its implications for $\mathcal{N}=1$ SYM theory has been performed by Loewy and Sonnenschein [14]. The importance of the gaugino condensate for de-singularizing the SUGRA solution was discussed by Apreda, Bigazzi, Cotrone, Petrini and Zaffaroni [15]. Building on this observation, Di Vecchia, Lerda and Merlatti [11, 16, 17] used a D5-brane embedded in the MN background and wrapping a certain two-cycle in order extract the running of the gauge coupling and the θ_{YM} angle. In particular, they found agreement to leading order with the Novikov-Shifman-Vainshtain-Zakharov (NSVZ) beta function [18, 19] and evidence for fractional instantons. However, their specific way of wrapping the D5-branes led to a number of problems, most notably, their full beta function contains terms, which are logarithmic in the coupling and cannot be interpreted in field theory. Although the two-loop coefficient of the beta function can be adjusted by changing the radial/energy relation [20], the problem of the logarithmic terms remained. Its resolution involves a suitable change of renormalization scheme, which could be translated to a change of the two-cycle around which the D5-brane is wrapped. In fact, Bertolini and Merlatti [21] solved this problem by wrapping the D5-brane on a different cycle, which had already been indicated in the paper by MN.

Other aspects of the MN solution, which have been studied in the literature, include the resolution of the conifold singularity using black holes [22], a different approach to the radial/energy relation [23], non-supersymmetric deformations [24, 25, 26], its non-commutative extension [27] and its Penrose (pp-wave) limit [28, 29, 30].

In this paper, we shall again consider the MN solution and add a number of new ingredients to the discussion. Thus, we hope not only to contribute to a better understanding of this specific case of the gauge/gravity duality, but also to provide a guideline for analyzing other cases.

Let us now summarize our paper and interpret the main results. One of the main ideas of the gauge/gravity duality is that there exists a dictionary relating the SUGRA fields to certain SYM operators. Taking the simplest approach, we generalize the MN solution by allowing for three integration constants, which have a precise physical meaning in the dual SYM theory. These are the following. First, we introduce a constant angle, ψ_0, by performing in the MN solution a global rotation of the frame of the twisted three-sphere. In the SYM theory, ψ_0 is identified with the phase of the gluino condensate. Moreover, it also determines the vacuum angle, θ_0, which is the value of θ_{YM} at the vacuum. For each θ_0, there are N inequivalent values of ψ_0 giving rise to the N physically inequivalent vacua. Second, the dilaton constant Φ_0 relates the value of the dynamically generated SYM scale Λ to the string parameters by the relation $\mathrm{e}^{2\Phi_0} = 2(2\pi)^4(\Lambda/L)^3/(N^3 g_s^2)$, where L is the SUGRA scale given by $L^{-2} = N g_s \alpha'$. This result stems from a direct calculation of the gluino condensate. Finally, we include an integration constant c, which can be found in the solution by Gubser, Tseytlin and Volkov [31], and which controls the resolution of the singularity. In fact, for $c > 0$, the bulk geometries possess a bad naked singularity, whereas the case $c = 0$ corresponds to the regular MN solution. The most natural interpretation of this constant is that it measures the amount of non-perturbative SYM physics captured by the dual SUGRA geometry. For $c = \infty$, the SUGRA solution contains only perturbative effects, whereas the MN solution describes also the non-perturbative physics. This interpretation is supported by the running of the gauge coupling, the value of the gluino condensate and the behaviour of the probe brane potential. The (generalized) MN solution is reviewed in section 2.

The MN solution is of the form $\mathbb{R}^{1,3} \times M_6$, where M_6 is a (non-compact) Calabi-Yau manifold. Its geometry encodes a variety of SYM quantities [32, 33, 34, 35]. Of these, we calculate in section 3 the gluino condensate and the effective superpotenial. The gluino condensate is a constant, which, for the regular MN solution, we identify with Λ^3, where Λ is the dynamically generated scale of the SYM theory [36].

The remaining sections deal with the application of the brane probe technique, which is laid out in section 4.[2] We obtain the gauge coupling and $\theta_{\rm YM}$ angle and, by considering the terms of the probe action that are independent of the gauge fields, the probe potential. Our analysis generalizes the calculation of [21], in that we do not fix the angular coordinate of the embedding. Thus, we obtain the correct (and in general non-vanishing) $\theta_{\rm YM}$. In contrast, the wrapping of [11] yields a $\theta_{\rm YM}$ differing from our result by a factor 1/2 (with a somewhat difficult interpretation of the chiral symmetry), while the result of [21] corresponds to the special case $\theta_{\rm YM} = 0$.

Section 5 focuses on the analysis of the gauge coupling obtained by the brane probe, but the breaking of the chiral symmetry by perturbative and non-perturbative effects ($\mathrm{U}(1) \to \mathbb{Z}_{2N} \to \mathbb{Z}_2$) shall also be discussed. The main result will be the calculation of the beta function. In contrast to [21] we shall average the probe gauge coupling over all inequivalent vacua in order to confront its running with a perturbative field theory analysis. This will not only remove a spurious energy dependence from $\theta_{\rm YM}$, but we will also be able to *exactly* re-write the beta function in terms of gauge theory quantities. It will turn out that the (singular) solution with $c = \infty$ correctly yields the complete perturbative running in terms the NSVZ beta function, and non-perturbative effects appear in terms of a (c-dependent) value of the gluino condensate, which might be re-written in terms of fractional instanton contributions in the far UV. Thus, our beta function predicts new terms, which have not been obtained in the field theory. In fact, the pole of the NSVZ beta function seems to disappear in the complete theory, which is dual to the (regular) MN solution. For the singular solutions, the location of the pole of the beta function coincides with the minimum of the effective superpotential from the Calabi-Yau geometry, which is thus interpreted as a vacuum-averaged result.

While the gauge coupling is relevant for the microscopic (UV) degrees of freedom, the probe potential provides a measure of the effective degrees of freedom around the vacuum. We shall analyze it in detail in section 6. For the singular solutions, the probe brane will fall into the singularity, and the state of lowest energy appears to be chirally invariant. Fortunately, the perturbative analysis of the coupling breaks down before the brane reaches the singularity. In contrast, for the regular case, the minimum of the probe

[2]The embedded object carrying the D5-brane action will be called a brane *probe*, although the usual zero-force condition cannot be imposed. Instead, we shall try to interpret the probe potential as an effective potential of the dual SYM theory.

potential is not chirally invariant, and we obtain a good description of the behaviour of the composite operator λ^2, where λ is the gluino field, around the vacuum. In the same region the probe potential will turn out to be closely related to the Veneziano-Yankielowicz effective potential [37].

2. Review of the MN solution

We consider the SUGRA solution corresponding to a system of N D5-branes. One way of finding it is by using $d = 7$ gauged SUGRA, which is obtained as a consistent truncation of the $d = 10$ SUGRA by compactification on an S^3 [38, 39]. This is a natural setting to incorporate the twist condition necessary to retain some supersymmetry [7, 8]. The metric in the string frame is [9, 11]

$$ds_{10}^2 = \mathrm{e}^{\Phi}\left[dx_{1,3}^2 + \frac{\mathrm{e}^{2h}}{L^2}\left(d\tilde{\theta}^2 + \sin^2\tilde{\theta}\, d\tilde{\phi}^2\right) + \frac{1}{L^2}d\rho^2 + \frac{1}{L^2}\sum_{a=1}^{3}(\sigma^a - LA^a)^2\right]. \qquad (2.1)$$

Here, $\tilde{\theta} \in [0, \pi)$ and $\tilde{\phi} \in [0, 2\pi)$ parameterize a two-sphere, S^2, which is part of the gauged SUGRA solution. The compactification three-sphere, S^3, is parameterized by the left-invariant one-forms σ^a $(a = 1, 2, 3)$ and is twisted by the SU(2) gauge field $A = \tau^a A^a$, where τ^a denote the Pauli matrices. For completeness, we give here the expressions for the σ^a,

$$\sigma^1 = \frac{1}{2}[\cos(\psi - \psi_0)\, d\theta + \sin(\psi - \psi_0)\sin\theta\, d\phi]\,, \qquad (2.2)$$

$$\sigma^2 = \frac{1}{2}[-\sin(\psi - \psi_0)\, d\theta + \cos(\psi - \psi_0)\sin\theta\, d\phi]\,, \qquad (2.3)$$

$$\sigma^3 = \frac{1}{2}[d\psi + \cos\theta\, d\phi]\,, \qquad (2.4)$$

which satisfy

$$d\sigma^a = -\varepsilon^{abc}\sigma^b \wedge \sigma^c\,. \qquad (2.5)$$

The angles are defined in the intervals $\phi \in [0, 2\pi)$, $\theta \in [0, \pi)$ and $\psi \in [0, 4\pi)$. In addition to the solution in the literature, we have included the constant angle ψ_0, which arises from a global U(1) gauge transformation. More precisely, one could consider the transformed gauge field $A' = g^{-1}Ag + ig^{-1}dg$, where $g \in \mathrm{SU}(2)$. Then, the part of the metric that belongs to the twisted 3-sphere can be written in the form (apart from the warp factor)

$$\sum_{a=1}^{3}(\sigma^a - LA'^a)^2 = \frac{1}{2}\,\mathrm{Tr}\,(\sigma - LA')^2 = \frac{1}{2}\,\mathrm{Tr}\,(g\sigma g^{-1} - iLdg\, g^{-1} - LA)^2\,. \qquad (2.6)$$

Hence, a *global* gauge transformation corresponds to a pure rotation of the frame on S^3, while *local* transformations will, in addition to the rotation, contribute to the twisting. Our frame is obtained from the MN frame by the transformation $g = \exp(-i\psi_0\tau^3/2)$.

The dilaton Φ and the prefactor e^{2h} in the metric are functions of the radial variable ρ and are given by

$$\begin{aligned} e^{2\Phi} &= e^{2\Phi_0} f(c) \frac{\sinh(2\rho + c)}{2e^h}\,, && (2.7)\\ e^{2h} &= \rho \coth(2\rho + c) - \frac{1}{4}[a(\rho)^2 + 1]\,, && (2.8)\\ a(\rho) &= \frac{2\rho}{\sinh(2\rho + c)}\,. && (2.9) \end{aligned}$$

In eq. (2.7), $f(c)$ is part of the overall constant, but we choose not to absorb it into Φ_0. The reason for this is that we want to consider c and Φ_0 as *independent* integration constants related to distict features of the dual gauge theory. We impose that for the MN solution ($c = 0$), $f(0) = 1$. Moreover, if the solution with $c = \infty$ and finite Φ_0 is to make sense, we also need $f(c) \sim e^{-c}$ for $c \to \infty$. $f(c)$ shall be determined in section 5.

The SU(2) gauge fields, A^a, are given by

$$A^1 = \frac{a(\rho)}{2L} d\tilde{\theta}\,, \qquad A^2 = \frac{a(\rho)}{2L} \sin\tilde{\theta}\, d\tilde{\phi}\,, \qquad A^3 = \frac{1}{2L} \cos\tilde{\theta}\, d\tilde{\phi}\,, \tag{2.10}$$

with the field strengths $F^a = dA^a + L\epsilon^{abc} A^b \wedge A^c$,

$$F^1 = \frac{\dot{a}(\rho)}{2L} d\rho \wedge d\tilde{\theta}\,, \qquad F^2 = \frac{\dot{a}(\rho)}{2L} d\rho \wedge \sin\tilde{\theta}\, d\tilde{\phi}\,, \qquad F^3 = \frac{1}{2L}\left[a(\rho)^2 - 1\right] d\tilde{\theta} \wedge \sin\tilde{\theta}\, d\tilde{\phi}\,. \tag{2.11}$$

The dot denotes a derivative with respect to ρ.

Furthermore, the solution contains a 2-form potential

$$\begin{aligned} C^{(2)} &= \frac{1}{4L^2}\left[\psi\left(\sin\theta\, d\theta \wedge d\phi) + \sin\tilde{\theta}\, d\tilde{\theta} \wedge d\tilde{\phi}\right) + \cos\theta \cos\tilde{\theta}\, d\phi \wedge d\tilde{\phi}\right] - \\ &\quad - \frac{a(\rho)}{2L^2}\left(d\tilde{\theta} \wedge \sigma^1 + \sin\tilde{\theta}\, d\tilde{\phi} \wedge \sigma^2\right), \end{aligned} \tag{2.12}$$

whose 3-form field strength $F^{(3)} = dC^{(2)}$ is

$$F^{(3)} = \frac{2}{L^2}\left(\sigma^1 - LA^1\right) \wedge \left(\sigma^2 - LA^2\right) \wedge \left(\sigma^3 - LA^3\right) - \frac{1}{L}\sum_{a=1}^{3} F^a \wedge \left(\sigma^a - LA^a\right). \tag{2.13}$$

The metric (2.1) is real for $\rho \geq \hat{\rho}$, where $\hat{\rho}$ is defined by $e^{2h(\hat{\rho})} = 0$. It is not difficult to show that $\hat{\rho}$ is implicitly determined by the transcedental equation

$$2\hat{\rho}\left[\coth(2\hat{\rho} + c) + 1\right] = 1\,, \tag{2.14}$$

which has a unique solution $0 \leq \hat{\rho} \leq 1/4$. The limiting cases are $\hat{\rho} = 0$ for $c = 0$ and $\hat{\rho} = 1/4$ for $c = \infty$.

The constant L is related to the number N of wrapped D5-branes by the usual charge quantization condition

$$\frac{1}{2\kappa_{10}^2}\int_{S^3} F^{(3)} = N\tau_5\,. \tag{2.15}$$

Using $\kappa_{10} = 8\pi^{7/2} g_s \alpha'^2$ and $\tau_5 = (2\pi)^{-5} g_s^{-1} \alpha'^{-3}$ one obtains [11]

$$L^{-2} = N g_s \alpha' \,. \tag{2.16}$$

In addition to the fields listed so far, there is a non-zero 6-form potential, $C^{(6)}$, defined by $dC^{(6)} = \star F^{(3)}$, where the Hodge dual is taken with respect to the string frame metric (2.1). Using eqs. (2.1), (2.10), (2.11) and (2.13), it is straightforward to obtain

$$\begin{aligned} dC^{(6)} &= -\frac{1}{L^2} v^{(4)} \wedge \Bigg\{ 2\mathrm{e}^{2\Phi} \left[\mathrm{e}^{2h} + \frac{1}{16} \mathrm{e}^{-2h} (1 - a^4) \right] d\rho \wedge d\tilde{\theta} \wedge \sin\tilde{\theta}\, d\tilde{\phi} \\ &\quad + \frac{1}{4} d \left[\mathrm{e}^{2\Phi} \dot{a} \left(d\tilde{\theta} \wedge \sigma^2 - \sin\tilde{\theta}\, d\tilde{\phi} \wedge \sigma^1 \right) - (a^2 - 1) \mathrm{e}^{2\Phi - 2h} d\rho \wedge (\sigma^3 - LA^3) \right] \Bigg\} \,, \end{aligned} \tag{2.17}$$

where we have abbreviated $v^{(4)} = dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4$. Thus, the 6-form potential is

$$\begin{aligned} C^{(6)} &= -\frac{1}{L^2} v^{(4)} \wedge \Bigg\{ \Psi(\rho)\, d\tilde{\theta} \wedge \sin\tilde{\theta}\, d\tilde{\phi} + \\ &\qquad + \frac{1}{4} \Big[\mathrm{e}^{2\Phi} \dot{a} \left(d\tilde{\theta} \wedge \sigma^2 - \sin\tilde{\theta}\, d\tilde{\phi} \wedge \sigma^1 \right) - \\ &\qquad\qquad - (a^2 - 1) \mathrm{e}^{2\Phi - 2h} d\rho \wedge \left(\sigma^3 - LA^3 \right) \Big] \Bigg\} \,, \end{aligned} \tag{2.18}$$

where the function $\Psi(\rho)$ satisfies

$$\dot{\Psi}(\rho) = 2\mathrm{e}^{2\Phi} \left[\mathrm{e}^{2h} + \frac{1}{16} \mathrm{e}^{-2h} (1 - a^4) \right] . \tag{2.19}$$

We were not able to integrate this equation, except for the case $c = \infty$, where $\Psi = \mathrm{e}^{2\Phi}(\rho - 1/2) + \Psi_0$, with Ψ_0 being an integration constant. We shall comment further on the function Ψ in section 6.

3. Gluino condensate and effective superpotential

Let us start our analysis by considering the geometry of the "internal" manifold. The bulk solution is—apart from the warp factor—of the form $\mathbb{R}^{1,3} \times M_6$, where M_6 is a Kähler manifold and geometrically encodes various aspects of the dual gauge theory. It encodes, first, the effective superpotential

$$W_{\text{eff}} \sim \int_{M_6} F^{(3)} \wedge \Omega \,, \tag{3.1}$$

where Ω is the holomorphic 3-form of the complex manifold M_6. Since M_6 is not compact, W_{eff} explicitly depends on a cut-off. The holomorphic 3-form Ω is given by [40, 41][3]

$$\Omega = \frac{\mathrm{e}^{2\Phi}}{L^3} \mathcal{E}^1 \wedge \mathcal{E}^2 \wedge \mathcal{E}^3 \,, \tag{3.2}$$

[3]We have chosen the phase so that W_{eff} is real.

where the complex 1-forms $\mathcal{E}^i$ are defined by

$$\begin{aligned}
\mathcal{E}^1 &= (\sigma^3 - LA^3) - id\rho\,, && (3.3)\\
\mathcal{E}^2 &= (\sigma^1 - LA^1) + iX\mathrm{e}^h \sin\tilde{\theta} d\tilde{\phi} - iP(\sigma^2 - LA^2)\,, && (3.4)\\
\mathcal{E}^3 &= \mathrm{e}^h d\tilde{\theta} + iX(\sigma^2 - LA^2) + iP\mathrm{e}^h \sin\tilde{\theta} d\tilde{\phi}\,, && (3.5)
\end{aligned}$$

and

$$P = \frac{\sinh(4\rho + 2c) - 4\rho}{2\sinh^2(2\rho + c)}\,, \qquad X = (1 - P^2)^{1/2} = \frac{2\mathrm{e}^h}{\sinh(2\rho + c)}\,. \tag{3.6}$$

A straightforward calculation yields

$$W_{\text{eff}}(\rho_0) \sim \frac{16\pi^3}{L^5}\mathrm{e}^{2\Phi_0} f(c) \int_{\hat{\rho}}^{\rho_0} d\rho\,[2\rho\coth(2\rho + c) - 1]\,. \tag{3.7}$$

The effective superpotential can be recast in terms of a pre-potential after introducing a canonical basis of homology 3-cycles of M_6 [33, 34],

$$W_{\text{eff}} \sim \int_A F^{(3)} \int_B \Omega - \int_A \Omega \int_B F^{(3)}\,. \tag{3.8}$$

In our case the compact 3-cycle is $A = S^3$, and the non-compact 3-cycle B has a complicated form. Since we have already found W_{eff}, we shall consider only the compact S^3. The integral of $F^{(3)}$ over S^3 is proportional to the number of D-branes, N, see eq. (2.15), while the integral of Ω over S^3 encodes the gluino condensate,

$$2\pi i|\langle\lambda^2\rangle_c| = \tau_5 \int_{S^3} \Omega = i\tau_5 \frac{2\pi^2}{L^3}\mathrm{e}^{2\Phi} X = i\tau_5 \frac{2\pi^2}{L^3}\mathrm{e}^{2\Phi_0} f(c)\,. \tag{3.9}$$

The constant τ_5 is needed for dimensional reasons. Following our interpretation of the integration constants, we re-write eq. (3.9) as

$$|\langle\lambda^2\rangle_c| = |\langle\lambda^2\rangle_0| f(c) = \Lambda^3 f(c)\,, \tag{3.10}$$

where we have used the convention $f(0) = 1$ and the fact that the regular solution is the true dual of $\mathcal{N} = 1$ SYM theory, i.e. $\langle\lambda^2\rangle_0 = \Lambda^3$, where Λ is the dynamically generated mass scale. Thus, we identify the precise role of Φ_0 relating Λ to the SUGRA parameters by the relation

$$\mathrm{e}^{2\Phi_0} = \pi^{-1}\tau_5^{-1}\Lambda^3 L^3 = \frac{2(2\pi)^4}{N^3 g_s^2}\left(\frac{\Lambda}{L}\right)^3\,. \tag{3.11}$$

Obviously, for $c = \infty$ we have $|\langle\lambda^2\rangle_\infty| = 0$, in agreement with the fact that a purely perturbative calculation fails to exhibit the gluino condensate.

4. Brane probe analysis

Another way of obtaining information about the dual field theory is by using the probe technique. Let us consider a D5-brane embedded in the background (2.1). Its action is given by

$$S = -\tau_5 \int d^6\xi \, \mathrm{e}^{-\Phi} \sqrt{-\det(G + 2\pi\alpha' F)} + \tau_5 \int \left(\sum_n C^{(n)} \wedge \mathrm{e}^{2\pi\alpha' F} \right)_{6\text{-form}} . \tag{4.1}$$

We consider a D5-brane wrapping a two-sphere parameterized by two angles $\hat{\theta}$ and $\hat{\phi}$. Expanding the Born-Infeld part of the action (4.1) and demanding that the non-abelian gauge fields F live only in the 4d part of the D5-branes, one finds

$$S = -\int d^4x \left[V + \frac{1}{4g_{\mathrm{YM}}^2} F_{\mu\nu}^A F_A^{\mu\nu} - \frac{\theta_{\mathrm{YM}} + 2\pi n}{32\pi^2} F_{\mu\nu}^A (\star F_A)^{\mu\nu} \right], \tag{4.2}$$

where the raising of the indices and the dual of the gauge fields are taken using the 4d Minkowski metric, and we have used the convention $\mathrm{tr}(T^A T^B) = \frac{1}{2}\delta^{AB}$ for the colour trace over the non-abelian generators. The potential V is given by

$$V = \tau_5 \int d\hat{\theta} d\hat{\phi} \left(\mathrm{e}^{-\Phi} \sqrt{-G} - C^{(6)}_{1234\hat{\theta}\hat{\phi}} \right). \tag{4.3}$$

For the gauge coupling, g_{YM}, and the theta angle, θ_{YM}, one obtains [11]

$$\begin{aligned} \frac{1}{g_{\mathrm{YM}}^2} &= 2\pi^2 \alpha'^2 \tau_5 \int d\hat{\theta} d\hat{\phi} \, \mathrm{e}^{-3\Phi} \sqrt{-\det G}\,, \\ \theta_{\mathrm{YM}} &= (2\pi)^4 \alpha'^2 \tau_5 \int d\hat{\theta} d\hat{\phi} \, C^{(2)}_{\hat{\theta}\hat{\phi}} \mod 2\pi\,. \end{aligned} \tag{4.4}$$

In eqs. (4.2) and (4.4) we have used the fact that the physics of Yang-Mills theory is periodic in the theta angle with period 2π, and we adopt the convention $\theta_{\mathrm{YM}} \in [0, 2\pi)$. The metric, the 2-form and the 6-form are induced from the respective bulk fields.

In order to proceed we have to specify how the world volume coordinates of the D5-brane are related to the bulk coordinates of the MN solution. The flat 4d part is obvious, but the wrapped S^2 needs some care. In order to use the coordinates ρ and ψ as parameters, we have to ensure that both of them are trivially fibred over the world volume [9]. This is done by imposing the four embedding conditions[4]

$$\theta = \tilde{\theta} = \hat{\theta}\,, \quad \phi = \tilde{\phi} = \hat{\phi}\,, \quad \psi = \mathrm{const}\,, \quad \rho = \mathrm{const}\,. \tag{4.5}$$

Thus, we have $d\rho = 0$ and $\sigma^3 - LA^3 = 0$ on the world volume. Notice that the first two conditions differ from the ones used in [11], where θ and ϕ are kept constant. Hence, the induced metric on the world volume of the D5-branes becomes

$$ds_6^2 = \mathrm{e}^{\Phi} \left\{ dx_{1,3}^2 + \frac{1}{4L^2} \left[4\mathrm{e}^{2h} + a(\rho)^2 + 1 - 2a(\rho)\cos(\psi - \psi_0) \right] \left(d\hat{\theta}^2 + \sin^2\hat{\theta}\, d\hat{\phi}^2 \right) \right\}, \tag{4.6}$$

[4] The alternative $\theta = -\tilde{\theta}$ is physically equivalent.

so that

$$\sqrt{-G} = e^{3\Phi} \sin\hat{\theta} \frac{1}{L^2} \left[\rho \coth(2\rho + c) - \frac{1}{2} a(\rho) \cos(\psi - \psi_0) \right] . \tag{4.7}$$

Moreover, the induced 2- and 6-forms are

$$C^{(2)} = \frac{1}{2L^2} \sin\hat{\theta}\, d\hat{\theta} \wedge d\hat{\phi} \, [\psi - a(\rho) \sin(\psi - \psi_0)] \, , \tag{4.8}$$

$$C^{(6)}_{1234\hat{\theta}\hat{\phi}} = -\frac{1}{L^2} \sin\hat{\theta} \left[\Psi(\rho) + \frac{1}{4} e^{2\Phi} \dot{a} \cos(\psi - \psi_0) \right] , \tag{4.9}$$

respectively.

Inserting these equations into the general expressions for V, $g_{\rm YM}$ and $\theta_{\rm YM}$, we find

$$V = \frac{4\pi\tau_5}{L^2} \left[e^{2\Phi} \rho \coth(2\rho + c) + \Psi(\rho) - \frac{1}{4} e^{2\Phi} (2a - \dot{a}) \cos(\psi - \psi_0) \right] , \tag{4.10}$$

as well as

$$\begin{aligned} \frac{1}{g^2_{\rm YM}} &= \frac{N}{4\pi^2} \left[\rho \coth(2\rho + c) - \frac{1}{2} a(\rho) \cos(\psi - \psi_0) \right] , \\ \theta_{\rm YM} &= N \left[\psi - a(\rho) \sin(\psi - \psi_0) \right] \quad \bmod 2\pi \, . \end{aligned} \tag{4.11}$$

5. Beta function

In this section we shall analyze and interpret the gauge coupling and $\theta_{\rm YM}$ angle measured by a probe D5-brane. Our main interest lies in the calculation of the perturbative beta function, but other aspects, such as the breaking of chiral symmetry from classically U(1) to $\mathbb{Z}_{2N}$ in the perturbative regime and to $\mathbb{Z}_2$ by non-perturbative effects, will also become transparent.

Let us start by discussing the chiral symmetry. We argue that the transformation $\delta\psi = -2\epsilon$, where $\epsilon \in [0, 2\pi)$, corresponds to a chiral transformation of the dual SYM theory. The perturbative physics is correctly captured by the solution with $c = \infty$, in which case eqs. (4.11) and (4.11) simplify to

$$\frac{1}{g^2_{\rm YM}} = \frac{N}{4\pi^2} \rho \qquad \text{and} \qquad \theta_{\rm YM} = N\psi \quad \bmod 2\pi \, . \tag{5.1}$$

Moreover, in this solution the non-abelian gauge field becomes abelian by virtue of $a(\rho) = 0$, which removes all terms with $\sin(\psi - \psi_0)$ and $\cos(\psi - \psi_0)$ from the metric and the form fields. Hence, the metric and the field strengths $dC^{(2)}$ and $dC^{(6)}$ are symmetric under $\delta\psi = -2\epsilon$ for all $\epsilon \in [0, 2\pi)$. These transformations form the classical chiral symmetry group U(1). However, $\theta_{\rm YM}$ is determined by $C^{(2)}$, which is not invariant under a general chiral transformation. In fact, eq. (5.1) is invariant only for $\epsilon = \pi(n-1)/N$ for $n = 1 \ldots 2N$, corresponding to the group $\mathbb{Z}_{2N}$ of the non-anomalous chiral symmetry transformations. In contrast, for every solution with $c < \infty$, where a certain amount of non-perturbative effects are included, terms with $\sin(\psi - \psi_0)$ and $\cos(\psi - \psi_0)$ appear showing that the symmetry of the bulk solutions is given by $\epsilon = \pi$ only, which represents the generator of the unbroken $\mathbb{Z}_2$ chiral symmetry of the quantum theory. We clearly see that the breaking $\mathbb{Z}_{2N} \to \mathbb{Z}_2$ is a non-perturbative effect.

JHEP02(2003)013

Let us turn now to the beta function. Considering the case $c = \infty$, which is given by eq. (5.1), one is tempted to identify $\mathrm{e}^{2\rho} \sim \mu^3$, where the exponent of μ is chosen such that the correct coefficient of the one-loop beta function is reproduced [15]. We shall present an alternative argument, which is applicable for any solution with $c < \infty$. To begin, let us combine g_{YM} and θ_{YM} to the complexified gauge coupling,

$$\tau = \frac{\theta_{\mathrm{YM}}}{2\pi} + i\frac{4\pi}{g_{\mathrm{YM}}^2} = i\frac{N}{2\pi}\left[2\rho\coth(2\rho + c) - i\psi - a(\rho)\mathrm{e}^{-i(\psi-\psi_0)}\right]. \tag{5.2}$$

In order to identify the energy scale μ, we follow [15, 11] and interpret the function $a(\rho)$ as the gluino condensate measured in units of μ,[5]

$$a(\rho) = \frac{|\langle\lambda^2\rangle_c|}{\mu^3}, \tag{5.3}$$

where $|\langle\lambda^2\rangle_c| = \Lambda^3 f(c)$ by virtue of eq. (3.10). We shall in the following be able to determine the function $f(c)$. The identification (5.3) is ambiguous in the case $c = \infty$, but the result of the following arguments will have a well-defined limit for $c \to \infty$, because any non-zero $|\langle\lambda^2\rangle_c|$ drops out when calculating the beta function.

The complex coupling (5.2) or, alternatively, eqs. (4.11) and (4.11), might be interpreted as exact, non-perturbative expressions. More precisely, starting at very large ρ and a certain initial ψ, the RG flow proceeds along the direction of steepest descent of the probe potential V (see section 6) towards the minimum at $\psi = \psi_0$, $\rho = \hat{\rho}$. In the regular case, $c = 0$, the gauge coupling, g_{YM}, diverges at $\rho = 0$, signalling the disappearance of the UV degrees of freedom at the vacuum. This exact RG flow was analyzed in [21] for the special case $\psi = \psi_0$.

The exact RG flow "knows" about the position of the vacuum at $\psi = \psi_0$ in the sense that the value of θ_{YM} flows towards $\theta_0 = N\psi_0 \mod 2\pi$. In contrast, perturbative calculations in field theory are typically ignorant about the vacuum state. For example, an argument as to why a direct perturbative calculation of the gaugino condensate yields zero is that the perturbative analysis averages over all vacua [42, 43]. We wish to confront our results with a perturbative field theory calculation and, therefore, we must average over all inequivalent vacua, which can be equivalently expressed as the following change of renormalization scheme,[6]

$$\tau \to \tau + i\frac{N}{2\pi}\frac{\langle\lambda^2\rangle_c}{\mu^3\mathrm{e}^{i\psi}}, \tag{5.4}$$

where we have written $\langle\lambda^2\rangle_c = |\langle\lambda^2\rangle_c|\mathrm{e}^{i\psi_0}$, leading to the coupling

$$\begin{aligned} \frac{1}{g_{\mathrm{YM}}^2} &= \frac{N}{4\pi^2}\rho\coth(2\rho + c), \\ \text{and the } \theta\text{-angle} \quad \theta_{\mathrm{YM}} &= N\psi \mod 2\pi. \end{aligned} \tag{5.5}$$

[5] A different approach to finding a radial/energy relation can be found in [23].

[6] It is irrelevant whether we average over the N discrete values of ψ_0 for a given θ_0, or whether we take the average over the whole interval.

It is interesting to observe that, in this renormalization scheme, the coupling (5.5) does not diverge for $\rho = c = 0$, in contrast to the exact coupling (4.11).

From eqs. (5.5) and (5.3) it is straightforward to obtain the beta function of the gauge coupling,

$$\beta = \mu \frac{dg_{\rm YM}}{d\mu} = -\frac{3N}{16\pi^2} g_{\rm YM}^3 \left[1 - a(\rho)^2 \frac{N g_{\rm YM}^2}{8\pi^2}\right] \left[1 - \frac{N g_{\rm YM}^2}{8\pi^2}\right]^{-1} . \tag{5.6}$$

For $c \to \infty$, where non-perturbative effects are absent, this coincides with the complete perturbative beta function of NSVZ [18, 19]. Notice also that in obtaining (5.6) no leading order approximation has been made in order to rewrite ρ in terms of $g_{\rm YM}$, and thus (5.6) holds at all energies. Non-perturbative effects are described by $a(\rho)$, which, in a large ρ expansion, takes the form of instanton corrections. In fact, $a(\rho) \approx 16\pi^2/(N g_{\rm YM}^2) \exp[-8\pi^2/(N g_{\rm YM}^2)] {\rm e}^{-c}$. We see that the constant c determines how much instanton physics is present in the running. Notice, however, that these non-perturbative effects have been obtained via a "perturbative" calculation, in that we are considering the vacuum-averaged coupling.

By using eq. (5.3), eq. (5.6) is recast in terms of gauge theory quantities only,

$$\beta = -\frac{3N}{16\pi^2} g_{\rm YM}^3 \left[1 - \frac{|\langle\lambda^2\rangle_c|^2}{\mu^6} \frac{N g_{\rm YM}^2}{8\pi^2}\right] \left[1 - \frac{N g_{\rm YM}^2}{8\pi^2}\right]^{-1} . \tag{5.7}$$

In this way we are also able to determine $f(c)$. In fact, consider the cases $c > 0$, where the beta function diverges for $g_{\rm YM}^2 = 8\pi^2/N$ or, equivalently, for $\rho = \rho_\Lambda$, where ρ_Λ is determined by

$$2\rho_\Lambda \coth(2\rho_\Lambda + c) = 1 . \tag{5.8}$$

Clearly, we have $\rho_\Lambda > \hat{\rho}$. For the limiting cases, $c = \infty$ and $c = 0$, we find $\rho_\Lambda = 1/2$ and $\rho_\Lambda = 0$, respectively. (The regular case $c = 0$, in which $\rho_\Lambda = \hat{\rho} = 0$, will be considered below.) Thus, the perturbative analysis breaks down before the brane reaches the singularity. Defining Λ as the scale where the beta function diverges, we obtain from eq. (5.3) that

$$f(c) = a(\rho_\Lambda) = \sqrt{1 - 4\rho_\Lambda^2} , \tag{5.9}$$

where ρ_Λ is determined implicitly by eq. (5.8). It is straightforward to check that $f(0) = 1$ and, for $c \to \infty$, $f(c) \approx 2{\rm e}^{-(c+1)}$, as needed.

In the regular case the beta function is not singular at $\rho = \rho_\Lambda = 0$, because also the numerator is zero. More precisely, the limit is

$$\beta \to -\frac{3N}{8\pi^2} g_{\rm YM}^3 = -3 \left(\frac{8\pi^2}{N}\right)^{1/2} . \tag{5.10}$$

Hence, the running changes from NSVZ type for very large μ to pure one-loop at $\mu = \Lambda$. Notice, however, the factor 2 with respect to the UV one-loop coefficient.

An interesting observation is that ρ_Λ coincides with the location of the minimum of the effective superpotential, $W_{\rm eff}$, cf. eq. (3.7). Thus, we are lead to interpret $W_{\rm eff}$ as a "perturbative" expression that averages over all inequivalent vacua.

JHEP02(2003)013

6. Probe potential

The vacuum of $\mathcal{N}=1$ SYM is usually described by the Veneziano-Yankielowicz effective potential, $S\log S$, where S is a chiral superfield containing the composite operator λ^2, where λ is the gluino field, as its lowest component. After integrating out the auxiliary fields, this becomes [37],

$$V_{\mathrm{VY}} \sim |\phi|^{4/3}\left[\ln^2\frac{|\phi|}{\Lambda^3}+\left(\alpha-\frac{\theta_0+2\pi n}{N}\right)^2\right]. \tag{6.1}$$

In the notation of [37], $\mu^3 = \Lambda^3 \mathrm{e}^{i\theta_0/N}$, where θ_0 is the vacuum angle. The integer n stems from the fact that we may choose a branch of the logarithm $\ln[(\phi/\mu^3)^N]$ [44]. The scalar field $\phi = |\phi|\mathrm{e}^{i\alpha} = \lambda^2$ describes the effective degrees of freedom at low energies. Since α has period 2π, there are N inequivalent values of n. Hence, the minimum of V_{VY} at $\phi = \Lambda^3\mathrm{e}^{i(\theta_0+2\pi n)/N}$ describes the N inequivalent vacua of $\mathcal{N}=1$ SYM. The chirally symmetric minimum at $|\phi|=0$ is unphysical, since the second derivative of V_{VY} diverges.

We would like to find a SUGRA derived quantity that can be compared to V_{VY}. The effective superpotential (3.7) is not a good candidate, because it does not contain an angular variable that could play the role of α. We shall argue in the following that the probe brane potential V can be compared to a potential derived from V_{VY} in the vicinity of the vacuum. Consider the potential

$$\tilde{V} = \left(\frac{\Lambda^3}{|\phi|}\right)^{\kappa} V_{\mathrm{VY}} = C\Lambda^{3\kappa}|\phi|^{4/3-\kappa}\left[\ln^2\frac{|\phi|}{\Lambda^3}+\left(\alpha-\frac{\theta_0+2\pi n}{N}\right)^2\right], \tag{6.2}$$

where C is a dimensionless constant. The potential $\tilde{V}$ possesses the same physical minimum at $|\phi| = \Lambda^3$ as V_{VY}, whereas the existence and the properties of the unphysical minimum at $\phi=0$ depend on κ. In fact, $\phi=0$ is a minimum of $\tilde{V}$, if $\kappa<4/3$, but for $1/3<\kappa<4/3$ the first derivative at this minimum is singular. We shall determine in the following that the probe potential V coincides with $\tilde{V}$ around the vacuum for $C = L/\Lambda$ and $\kappa=1$.

The brane potential V is given by eq. (4.10),

$$V = \frac{4\pi\tau_5}{L^2}\left[\mathrm{e}^{2\Phi}\rho\coth(2\rho+c)+\Psi(\rho)-\frac{1}{4}\mathrm{e}^{2\Phi}(2a-\dot{a})\cos(\psi-\psi_0)\right], \tag{6.3}$$

where the function Ψ satisfies eq. (2.19)

$$\dot{\Psi}(\rho) = 2\mathrm{e}^{2\Phi}\left[\mathrm{e}^{2h}+\frac{1}{16}\mathrm{e}^{-2h}\left(1-a^4\right)\right]. \tag{6.4}$$

In order to discuss V, we must again distinguish between the singular solutions, $c>0$, and the regular one, $c=0$, which are qualitatively very different. Let us start with the singular cases. We were not able to integrate eq. (6.4) except for the case $c=\infty$, where $\Psi = \mathrm{e}^{2\Phi}(\rho-1/2)+\Psi_0$. The integration constant Ψ_0 plays the role of a zero-point energy, and we shall set it to zero ensuring $V=0$ for $\rho=\hat{\rho}=1/4$. Hence, we obtain

$$V_{c=\infty} = 4L\Lambda^3\mathrm{e}^{2\rho-1}\sqrt{\rho-\frac{1}{4}}, \tag{6.5}$$

where we have used eq. (3.11) to eliminate Φ_0 and eq. (5.9) for $f(c)$. Eq. (6.5) captures not only the large ρ behaviour of V in all cases, but its behaviour at the singularity is typical for the cases with $c>0$. In fact, making the ansatz $\Psi = \mathrm{e}^{2\Phi}[\rho - 1/2 + f(\rho)]$, we find from eq. (6.4) the following differential equation for f,

$$\frac{d}{d\rho}\left(\frac{\rho f}{a\mathrm{e}^h}\right) = -\frac{\rho}{a\mathrm{e}^{3h}}\left(\mathrm{e}^{2h} + \frac{a^2-1}{4}\right)\left[1 - \frac{1}{\rho}\left(\mathrm{e}^{2h} - \frac{a^2-1}{4}\right)\right]. \tag{6.6}$$

It is not difficult to show that f behaves as

$$f = f_0\sqrt{\rho - \hat{\rho}} + \frac{1-4\hat{\rho}}{\hat{\rho}}(\rho - \hat{\rho}) + \cdots \tag{6.7}$$

close to the singularity. Setting the integration constant f_0 to zero we ensure again $V=0$ for $\rho = \hat{\rho}$. Moreover, using $\mathrm{e}^{2\Phi} \sim (\rho-\hat{\rho})^{-1/2}$ and $2a - \dot{a} \sim \rho - \hat{\rho}$, we obtain $V \sim \sqrt{\rho - \hat{\rho}}$ with a positive proportionality constant close to the singularity. Hence, the potential has its absolute minimum at $\rho = \hat{\rho}$, but the brane probe feels an infinite attractive force at the singularity. Furthermore, the last term in eq. (6.3) containing $\cos(\psi - \psi_0)$ exactly vanishes at the singularity, which means that the state of lowest potential energy does not depend on the choice of ψ. Both features of the minimum—invariance under variations of ψ and a singular first derivative—are in common with the unphysical minimum of the potential $\tilde{V}$, although the comparison does not stand up to a more quantitative analysis.

We shall discuss now the regular solution and find a quantitative agreement between V and $\tilde{V}$ close to the vacuum for $\kappa = 1$ and $C = L/\Lambda$. The first feature, which is different from the singular cases, is that the coefficient in V of $\cos(\psi - \psi_0)$ is strictly negative. Hence, the vacuum must be found at $\psi = \psi_0$. (Any multiples of 2π are irrelevant.) From eq. (4.11) we now have that $\psi_0 = (\theta_0 + 2\pi n)/N$, so that we can re-write $\cos(\psi - \psi_0)$ as $\cos[\psi - (\theta_0 + 2\pi n)/N]$, where θ_0 is the field theory vacuum angle. Expanding the cosine in V to quadratic order and comparing it with $\tilde{V}$, we find that $\psi = \alpha$, and we can make the following identifications, which we expect to hold in the vicinity to the physical minimum,

$$C\Lambda^{3\kappa}|\phi|^{4/3-\kappa} \approx \frac{4\pi\tau_5}{L^2}\mathrm{e}^{2\Phi}\frac{1}{8}(2a - \dot{a}), \tag{6.8}$$

$$C\Lambda^{3\kappa}|\phi|^{4/3-\kappa}\ln^2\frac{|\phi|}{\Lambda^3} \approx \frac{4\pi\tau_5}{L^2}\left[\mathrm{e}^{2\Phi}\left(\rho\coth(2\rho) - \frac{1}{2}a + \frac{1}{4}\dot{a}\right) + \Psi\right]. \tag{6.9}$$

It is straightforward to show that the right hand side of eq. (6.9) has exactly one local minimum, which is at $\rho = 0$. Thus, the potential V has its minimum at $\rho = 0$ and $\psi = \psi_0$, which is not chirally invariant.

Expanding eq. (6.8) about $\rho = 0$ and substituting eq. (3.11) for Φ_0, we find

$$C\Lambda^{3\kappa}|\phi|^{4/3-\kappa} = L\Lambda^3\left(1 + \frac{2}{3}\rho + \frac{2}{9}\rho^2 + \mathcal{O}\left(\rho^3\right)\right). \tag{6.10}$$

Since $|\phi| = \Lambda^3$ at the minimum, we obtain $C = L/\Lambda$. Then, from eq. (6.10) follows

$$\ln\frac{|\phi|}{\Lambda^3} = \frac{3}{4-3\kappa}\left(\frac{2}{3}\rho + \mathcal{O}\left(\rho^3\right)\right),$$

which we use to expand the left hand side of eq. (6.9), thus obtaining

$$\frac{4\rho^2 L\Lambda^3}{(4-3\kappa)^2}\left(1+\frac{2}{3}\rho+\cdots\right)=\frac{4\pi\tau_5}{L^2}\mathrm{e}^{2\Phi_0}\left(\rho^2+\frac{2}{3}\rho^3+\cdots\right). \tag{6.11}$$

After substituting the constant Φ_0 we find agreement of the second and the third derivatives of the potentials V and $\tilde{V}$ at the minimum, if $\kappa=1$. (The fourth derivatives do not agree). Naively one might have expected to find $\kappa=0$, but one should bear in mind that, while V_{VY} is constructed from a holomorphic superfield, the probe potential is not intrinsically holomorphic.

Finally, we would like to point out that the result $\kappa=1$ is supported also by a consideration of the kinetic term. In fact, it is not difficult to show that, if we allow for small fluctuations $\rho(x)$ and $\psi(x)$ of the brane positions, the probe brane action gives rise also to the kinetic term

$$S_{\mathrm{kin}}=-\frac{2\pi\tau_5}{L^4}\int d^4x\,\mathrm{e}^{2\Phi}\left[\rho\coth(2\rho+c)-\frac{1}{2}a(\rho)\cos(\psi-\psi_0)\right]\left[(\partial\rho)^2+\frac{1}{4}(\partial\psi)^2\right]. \tag{6.12}$$

We consider the regular case, $c=0$, and evaluate the kinetic term in the vicinity of the vacuum. Up to quadratic order in ρ and $\psi-\psi_0$, we find

$$S_{\mathrm{kin}}\approx-\frac{2\Lambda^3}{L}\int d^4x\left[\rho^2+\frac{1}{4}(\psi-\psi_0)^2\right]\left[(\partial\rho)^2+\frac{1}{4}(\partial\psi)^2\right]. \tag{6.13}$$

Using $\kappa=1$ and $C=L/\Lambda$, we find to leading order from eq. (6.10) that $\rho\approx(|\phi|/\Lambda^3-1)/2$, so that eq. (6.13) can be re-written (again exact to quadratic order) as

$$S_{\mathrm{kin}}\approx-\frac{1}{8L\Lambda^9}\int d^4x\left(\phi-\Lambda^3\mathrm{e}^{i\psi_0}\right)\left(\phi^*-\Lambda^3\mathrm{e}^{-i\psi_0}\right)\partial_\mu\phi\partial^\mu\phi^*, \tag{6.14}$$

where $\phi=|\phi|\mathrm{e}^{i\psi}$. In order to obtain this complexified form the value $\kappa=1$ is crucial.

It would be very interesting to recast eq. (6.14) in terms of a Kähler potential.

Acknowledgments

I am indebted to A. Lerda for his collaboration on this topic and careful reading of the manuscript. Moreover, thanks go to M. Bertolini, P. Di Vecchia, A. Liccardo, R. Marotta, P. Merlatti, F. Nicodemi, C. Nuñez and R. Russo, as well as the other participants of the "gauge/gravity" workshop during the 2003 RTN Torino winter school, for useful discussions. Financial support by the European Commission under the contract RTN-2000-00131 and by a fellowship from INFN is gratefully acknowledged.

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JHEP02(2003)013

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